

Extremal Graph Theory

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Theorem 1 (Ramsey). *Every graph on*

$$n \geq \binom{a+b-2}{a-1}$$

vertices contains either a clique of size a or an independent set of size b .

Theorem 2 (Schur). *Any coloring of the integers with finitely many colors contains a monochromatic arithmetic progression with at least three terms.*

Theorem 3 (Mantel). *In a triangle-free graph, the number of edges in a graph with n vertices is at most $n^2/4$.*

Theorem 4. *The number of triangles in a graph with n vertices and m edges is at least*

$$\frac{4m^2}{3n} - \frac{mn}{3}.$$

Theorem 5 (Turán). *In a K_r -free graph, the edge density is at most*

$$1 - \frac{1}{r} + o(1).$$

Theorem 6 (Kővári-Sós-Turán). *In a $K_{s,t}$ -free graph, the edge density is $O(n^{-1/s})$.*

Theorem 7 (Erdős-Stone). *In a H -free graph, the edge density is at most*

$$1 - \frac{1}{\chi(H) - 1} + o(1).$$